UNSTEADY MOISTURE TRANSFER IN CAPILLARY-POROUS MATERIAL

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This paper gives the results of an experimental investigation of unsteady moisture transfer in chromatographic paper and compares them with the solution of the parabolic transport equation in Stefan's formulation.

Tanaeva in [1] attempted to modify the transport equation by the introduction of a term due to relaxation effects. This modification was required to make the results of analytical calculation agree with the experimental results.

We think that no modification should be made to the transport equation itself, but the boundary conditions should be formulated differently. Allowance must be made for the fact that some of the moisture may be bound by the porous material and will not be involved in the subsequent transfer process, just as in the case of melting some of the heat is absorbed on the phase interface.

If the coordinate origin is placed at the boundary of contact of the porous material and liquid, the boundary conditions for the transport equation

$$\frac{\partial U}{\partial \tau} = D\frac{\partial^2 U}{\partial x^2} \tag{1}$$

can be written in the form

$$U|_{x=h} = 0, \tag{2}$$

$$-D\left.\frac{\partial U}{\partial x}\right|_{x=h} = r\,\frac{dh}{d\tau}, \tag{3}$$

$$U|_{x=0} = U_c = \text{const.} \tag{4}$$

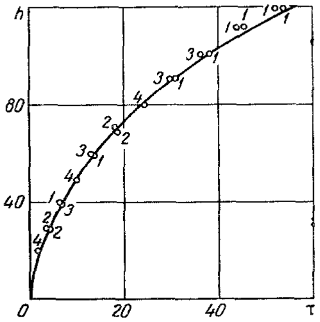

Fig. 1. Height of capillary rise of liquid h, mm as function of time τ, min. The figures beside the points indicate the number of experiments. Curve calculated from Eq. (6).

The formulated problem is a special case of Stefan's well-known problem, the solution of which can be put in the form [2]

$$U = U_c\left[1 - \frac{1}{\operatorname{erf}\mu}\operatorname{erf}\left(\mu\frac{x}{h}\right)\right], \tag{5}$$

$$h = \beta\sqrt{\tau}, \tag{6}$$

$$\mu = \beta/2\sqrt{D}, \tag{7}$$

$$\frac{U_c}{r}\exp[-\mu^2] = \sqrt{\pi}\,\mu\operatorname{erf}\mu. \tag{8}$$

It is well known that the phenomenological nature of the relationship between mass (heat) flow and concentration (temperature) gradient leads to an infinite velocity of propagation of mass (heat), although it is obviously finite in fact. We also know that the transport equation in the case where mass (heat) sources or sinks are present on the moving boundary leads to a finite velocity of advance of this boundary [2].

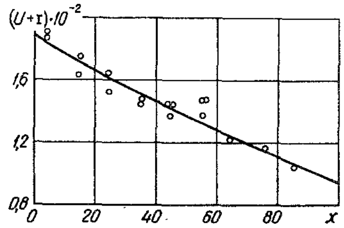

Fig. 2. Value of U + r as function of coordinate x, mm. Curve calculated from Eq. (5).

From the expression (5) we can easily obtain the integral value U_Σ:

$$U_\Sigma = \int_0^h U\,dx = hU_c \times$$

$$\times\left\{1 - \frac{1}{\operatorname{erf}\mu} + \frac{1}{\mu\operatorname{erf}\mu}\left[\operatorname{ierfc}0 - \operatorname{ierfc}\mu\right]\right\} = hU_c k. \tag{9}$$

Expressions (5) and (9) contain only two constant parameters, U_c and μ, which can be determined experimentally.

To test the hypothesis we carried out experiments with FN-1 (Filtrak) chromatographic paper. The choice of this as a capillary-porous material was based on the desire to avoid a number of errors associated with the use of loose materials (change in pore size in relation to method of loading and tamping, effect of walls of vessel, etc.). In the experiments we used suspended strips of chromatographic paper, 30 mm wide and 150 mm long, on which marks had been made at different heights.

In the experiments we determined:

1) the time for the liquid front to reach each mark;

2) the moisture content in 10 × 30 mm portions of the paper over the height of the specimen when the moisture front reached the 100-mm mark;

3) the total moisture content of the paper strips when the moisture front reached the 30-, 60-, 80-, 100-, and 120-mm marks.

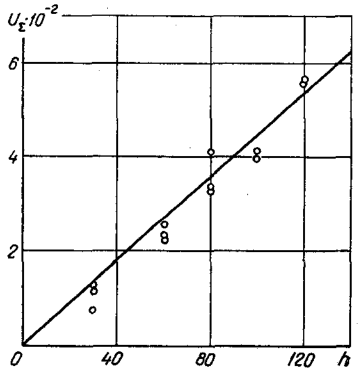

Fig. 3. Total excess moisture content U_Σ as function of h, mm. Curve calculated from Eq. (9).

The moisture content was determined by weighing in bottles on an analytical balance. We determined the weight of the moist paper and its weight after drying at 95°–100° C (to constant weight). We took measures to prevent evaporation of the liquid from the surface of the paper during the experiment. For this purpose the vessel containing the liquid and the strip of paper was covered with a clear plastic bell. We took into account the presence of hygroscopic moisture in the paper. The portion of the paper immersed in the liquid during the experiment was cut off. In measuring the local moisture content of the paper we cut the 10 × 30 mm strips of paper into appropriate parts after the experiment. A comparison of the results of calculation and the experimental data is shown in Figs. 1–3.

The experimentally obtained relationship between h and τ (Fig. 1) was in good agreement with Eq. (6) for $\beta = 1.63$ cm/min$^{1/2}$ (solid line). The relationship between the total moisture content U + r in different portions of the paper and the coordinate x with h = 100 mm (Fig. 2) was represented satisfactorily by Eq. (5) with $U_c = 1.90$ g/cm^2 and $r = 0.95$ g/cm^2. The value of μ, determined from Eq. (8), was 0.62 and the value of the coefficient D from Eq. (7) was 1.79 cm^2/min.

As Fig. 3 shows, the experimentally obtained relationship between U_Σ and h agreed fairly well with the theoretical straight line calculated from Eq. (9) with the parameters μ and U_c determined in the previous experiment.

These data indicate that if allowance is made for the binding of some of the moisture in the passage of liquid into porous material, at least for chromatographic paper, there is no need to modify the well-known parabolic equation.

This problem is analogous to the heat problems involving melting and solidification of substances.

NOTATION

τ) time; x) coordinate; U) specific excess moisture content; r) specific bound moisture content; D) diffusion coefficients; h) height of rise of moisture; β) moisture penetration coefficient; U_Σ) total excess moisture content of paper.

REFERENCES

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